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Minimagnetospheric Plasma Propulsion for Outer Planet Missions

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Introduction

THE basic idea of minimagnetospheric plasma propulsion (M2P2) is to create a large magnetic bubble around the spacecraft and to use the interaction between this artificial magnetic field and the solar wind to obtain momentum and increase the spacecraft thrust [1]. Although the possibility of realizing large and stable magnetic field inflation has not yet fully proved, various research groups are intensively working on this subject both numerically and experimentally [2,3].

From a theoretical viewpoint the M2P2 has significant differences against other low-thrust propulsion means and, in particular, it represents a promising option for short-term deep space missions requiring a large amount of ΔV [3]. The strength of M2P2 with respect to electric propulsion is based on the possibility to achieve higher thrust/power ratio than the ion engine. When compared with a solar sail, the main advantage of M2P2 is that the deployment phase occurs through electromagnetic processes and does not require large mechanical structures. Another difference is that the M2P2 can be easily turned off, thus allowing the presence of coast arcs in the spacecraft trajectory.

In spite of these distinctive characteristics, a thorough study of trajectories for outer planet missions using M2P2 has found little application until recently [4,5]. In their analysis, both Trask et al. [5] and Mengali and Quarta [4] assume a planar model and a circle-to-circle interplanetary transfer. However, using real ephemeris data for the departure and arrival planets, significant differences with respect to the two-dimensional case may occur both in terms of ΔV and in the flight time. The aim of this Note is to quantify such differences in a systematic framework, by developing the optimal control law that minimizes the impulsive velocity change required to accomplish the rendezvous between the spacecraft and the target planet. In particular, an Earth–Jupiter transfer is used as a representative mission candidate for a M2P2 application.

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Problem Formulation

The equations of motion for a spacecraft in a heliocentric inertial frame $\mathcal{T}_\odot(x, y, z)$ are

$$\dot{\boldsymbol{r}} = \boldsymbol{v} \tag{1}$$

$$\dot{\boldsymbol{v}} = -\frac{\mu_\odot}{r^3}\boldsymbol{r} + \frac{\tau F}{m}\hat{\boldsymbol{a}} \tag{2}$$

$$\dot{m} = -\tau\beta \tag{3}$$

where $[\boldsymbol{r}]_{\mathcal{T}_\odot} = [r_x, r_y, r_z]^T$ and $[\boldsymbol{v}]_{\mathcal{T}_\odot} = [v_x, v_y, v_z]^T$ are the spacecraft position and velocity relative to $\mathcal{T}_\odot$, $r \triangleq |\boldsymbol{r}|$ is the heliocentric distance of the spacecraft from the Sun, $\mu_\odot$ is the Sun's gravitational parameter, m is the spacecraft mass, F is the modulus of the thrust vector, $\hat{\boldsymbol{a}}$ is the thrust unit vector, $\tau = (0, 1)$ is the thruster switching function, and β is the propellant mass flow rate.

The M2P2 dipole axis, whose function is to generate the magnetosphere, can be tilted by a prescribed angle $\alpha \in [0, \alpha_{\max}]$ from the radial direction $\hat{\boldsymbol{r}} \triangleq \boldsymbol{r}/r$ to provide an azimuthal thrust. In other terms, the thrust vector is constrained to remain inside a cone whose half-opening angle is equal to $\alpha_{\max}$. Although the value of $\alpha_{\max}$ is rather small, on the order of 5 deg [1], the azimuthal thrust gives the spacecraft an additional maneuver capability in that it allows the spacecraft to perform three-dimensional trajectories and to leave the initial orbital plane.

The dipole axis rotation causes both an increase in the thrust vector modulus F and in the mass flow rate β. Paralleling the approach taken in a two-dimensional framework [4], we assume that both F and β are increasing functions of α. Because F remains constant as the thrust direction varies on the surface of a cone, the angle α will be referred to as cone angle. The value of α is obtained via

$$\alpha = \arccos(\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{r}}) \tag{4}$$

Let $\mathcal{T}_{\text{orb}}(x_{\text{orb}}, y_{\text{orb}}, z_{\text{orb}})$ be an orbital frame whose unit vectors are $\hat{\boldsymbol{i}}_{\text{orb}} \equiv \hat{\boldsymbol{r}}$, $\hat{\boldsymbol{j}}_{\text{orb}}$ and $\hat{\boldsymbol{k}}_{\text{orb}}$. Assume that the plane $z_{\text{orb}} = 0$ contains the axis z of the $\mathcal{T}_\odot$ frame and y_{orb} points toward the ecliptic pole. Then, the components of $\hat{\boldsymbol{a}}$ can be expressed in the $\mathcal{T}_{\text{orb}}$ frame as a function of the cone angle α and of the azimuth angle $\delta \in [-\pi, \pi]$ as

$$[\hat{\boldsymbol{a}}]_{\mathcal{T}_{\text{orb}}} = [\cos\alpha, \sin\alpha\cos\delta, \sin\alpha\sin\delta]^T \tag{5}$$

As a consequence, the system control variables are given by τ, α, and δ.

The low-thrust level and the moderate maneuver capabilities of an M2P2 thruster (recall that the thrust is nearly radial), do not allow this system to perform interplanetary rendezvous missions without the aid of another propulsive means. A typical solution [5] consists in adding a chemical propulsion system to provide the impulsive thrust necessary to set to zero the relative velocity between the spacecraft and the target and perform the rendezvous maneuver. The M2P2 is used during the transfer phase to drive the spacecraft near the target in an optimal manner, that is, in such a way to minimize the impulsive ΔV necessary to complete the rendezvous maneuver.

From a mathematical standpoint the problem can be translated into an optimal formulation as follows: find the time histories of the control variables τ, α, and δ that maximize the functional

$$J \triangleq -\Delta V^2 = -(\boldsymbol{v}_f - \boldsymbol{v}_{p_f}) \cdot (\boldsymbol{v}_f - \boldsymbol{v}_{p_f}) \tag{6}$$

where $\boldsymbol{v}_f$ and $\boldsymbol{v}_{p_f}$ are the spacecraft and target velocities, respectively, calculated at the rendezvous instant t_f.

Optimal Control Law

Recalling the equation of motion (1–3) the Hamiltonian of the system is

$$H = \boldsymbol{\lambda}_r \cdot \boldsymbol{v} + \boldsymbol{\lambda}_v \cdot \left[-\frac{\mu_\odot \boldsymbol{r}}{(\boldsymbol{r} \cdot \boldsymbol{r})^{3/2}} + \frac{\tau F}{m} \hat{\boldsymbol{a}} \right] - \lambda_m \tau \beta \tag{7}$$

where $\boldsymbol{\lambda}_r \triangleq [\lambda_{r_x}, \lambda_{r_y}, \lambda_{r_z}]^T$ and $\boldsymbol{\lambda}_v \triangleq [\lambda_{v_x}, \lambda_{v_y}, \lambda_{v_z}]^T$ are the vectors adjoint to the position and the velocity, respectively, and λ_m is the mass costate. Their time derivatives are provided by the Euler–Lagrange equations:

$$\dot{\boldsymbol{\lambda}}_r = -\frac{\partial H}{\partial \boldsymbol{r}} = \mu_\odot \frac{\boldsymbol{\lambda}_v - 3\hat{\boldsymbol{r}}(\hat{\boldsymbol{r}} \cdot \boldsymbol{\lambda}_v)}{r^3} \tag{8}$$

$$\dot{\boldsymbol{\lambda}}_v = -\frac{\partial H}{\partial \boldsymbol{v}} = -\boldsymbol{\lambda}_r \tag{9}$$

$$\dot{\lambda}_m = -\frac{\partial H}{\partial m} = \frac{\tau F(\boldsymbol{\lambda}_v \cdot \hat{\boldsymbol{a}})}{m^2} \tag{10}$$

Invoking Pontryagin's maximum principle, the optimal control law can be obtained by maximizing, at any time, that portion H' of the Hamiltonian that explicitly depends on the control variables:

$$H' \triangleq \frac{\tau F \lambda_v}{m} \left(\hat{\boldsymbol{\lambda}}_v \cdot \hat{\boldsymbol{a}} - \frac{\lambda_m m \beta}{F \lambda_v} \right) \tag{11}$$

where $\lambda_v = |\boldsymbol{\lambda}_v|$ and $\hat{\boldsymbol{\lambda}}_v = \boldsymbol{\lambda}_v / \lambda_v$ is the direction of the primer vector [6]. Equation (11) shows that the optimal clock angle δ is found by maximizing the projection of the thrust unit vector $\hat{\boldsymbol{a}}$ along the direction of the primer vector. The components of $\hat{\boldsymbol{\lambda}}_v$ in the $\mathcal{T}_{\text{orb}}$ frame can be suitably expressed in terms of the primer vector cone angle α_v and clock angle δ_v [7]. Following the same arguments used in [7], it can be easily verified that the maximization of H' occurs when $\tan\delta = \tan\delta_v$ and the unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{a}}$, and $\hat{\boldsymbol{\lambda}}_v$ are coplanar. As a consequence, $\hat{\boldsymbol{a}}$ can be written as

$$\hat{\boldsymbol{a}} = \frac{\sin(\alpha_v - \alpha)}{\sin\alpha_v} \hat{\boldsymbol{r}} + \frac{\sin\alpha}{\sin\alpha_v} \hat{\boldsymbol{\lambda}}_v \quad \text{for } \alpha_v \in (0, \pi) \tag{12}$$

where

$$\alpha_v \triangleq \arccos(\hat{\boldsymbol{\lambda}}_v \cdot \hat{\boldsymbol{r}}) \tag{13}$$

The optimal control law for the thruster switching function is found observing that H' depends linearly on τ. As a result, a bang-bang control is optimal:

$$\tau = \frac{1 + \operatorname{sign}(H')}{2} \tag{14}$$

where sign(.) is the signum function.

Finally, consider the problem of finding the optimal value of the cone angle $\alpha \in [0, \alpha_{\max}]$. Because $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{a}}$, and $\hat{\boldsymbol{\lambda}}_v$ are coplanar, one has

$$\hat{\boldsymbol{\lambda}}_v \cdot \hat{\boldsymbol{a}} = \cos(\alpha_v - \alpha) \tag{15}$$

Substituting Eq. (15) into (11) and imposing the necessary condition $\partial H'/\partial\alpha = 0$, the optimal value of α depends on the primer vector cone angle α_v through the following relationship:

$$\alpha_v = \alpha + \arctan\left(\frac{\mathcal{A}F - F_\alpha \sqrt{F^2 + F_\alpha^2 - \mathcal{A}^2}}{\mathcal{A}F_\alpha + F\sqrt{F^2 + F_\alpha^2 - \mathcal{A}^2}} \right) \tag{16}$$

where $\beta_\alpha \triangleq \partial\beta/\partial\alpha$, $F_\alpha \triangleq \partial F/\partial\alpha$ and

$$\mathcal{A} \triangleq \frac{\lambda_m m \beta_\alpha}{\lambda_v} \tag{17}$$

Once both the thrust modulus and the mass flow rate variations with the cone angle have been defined [that is, $F = F(\alpha)$ and $\beta = \beta(\alpha)$], Eq. (16) can be solved numerically for α as a function of α_v and $\mathcal{A}$. The dependence of both the thrust modulus and the mass flow rate on α can be suitably modeled through second order polynomial approximations, as suggested in [4]. Note that if β is nearly independent of α, then $\partial\beta/\partial\alpha \simeq 0$ and $\mathcal{A} \simeq 0$. In that case the optimal cone angle reduces to

$$\alpha_v \simeq \alpha - \arctan\left(\frac{F_\alpha}{F} \right) \tag{18}$$

The boundary conditions of the differential problem (1–3) and (8–10) are given by [8]

$$\boldsymbol{r}(t_0) = \boldsymbol{r}_\oplus(t_0), \qquad \boldsymbol{v}(t_0) = \boldsymbol{v}_\oplus(t_0), \qquad m(t_0) = m_0 \tag{19}$$

$$\boldsymbol{r}(t_f) = \boldsymbol{r}_p(t_f), \qquad \boldsymbol{\lambda}_v(t_f) = -2[\boldsymbol{v}(t_f) - \boldsymbol{v}_p(t_f)], \qquad \lambda_m(t_f) = 0 \tag{20}$$

where subscripts 0 and f refer to the initial and final condition, respectively, while $\oplus$ and p are the starting and arrival planets.

When the flight time is left free, the optimal mission time is obtained by enforcing the transversality condition $H(t_f) = 0$ [8].

Numerical Simulations

A set of canonical units [9] have been used in the integration of the differential equations to reduce their numerical sensitivity. The differential equations have been integrated in double precision using a Runge–Kutta fifth-order scheme with absolute and relative errors of 10^{-10}. For illustrative purposes the control law described before has been applied to study Earth–Jupiter missions. The optimal trajectories have been obtained using planetary orbital elements referenced to mean ecliptic and equinox of J2000 at the J2000 epoch [9]. The spacecraft position on the initial orbit is defined through the true anomaly ν_0 with respect to the perihelion. The final Jovian position (defined by its true final anomaly ν_f) is not fixed a priori, rather it is an output of the optimization process. This allows one not only to calculate the optimal ΔV, but also the optimal mission time and final position as a function of the value of the initial true anomaly ν_0. In other terms, the results obtained represent the global minimum solutions for Earth–Jupiter three-dimensional missions.

The control problem is remarkably simplified under the assumption that, during the whole transfer, the optimal value of the

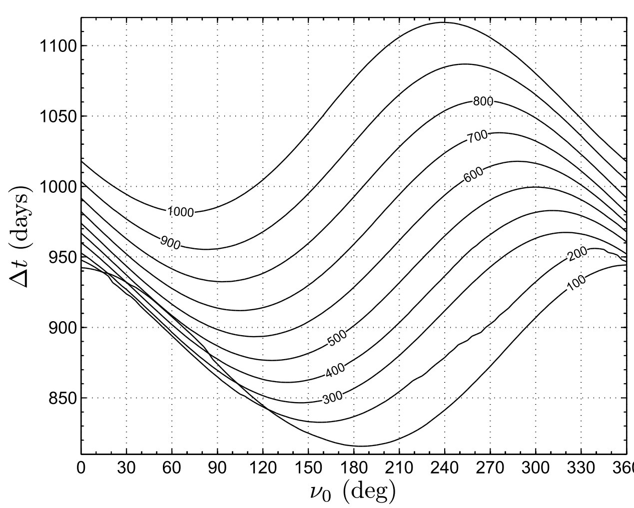

Fig. 1 Mission times, parametrized with m_0 (kg), as a function of the initial true anomaly of Earth.

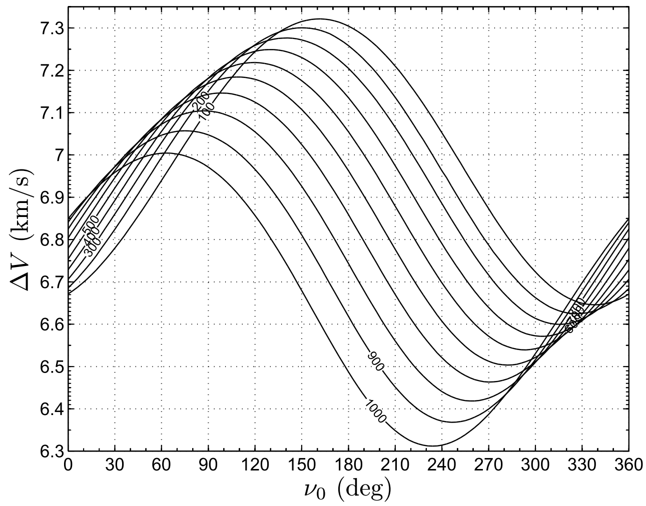

Fig. 2 Required ΔV, parametrized with m_0 (kg), as a function of the initial true anomaly of Earth.

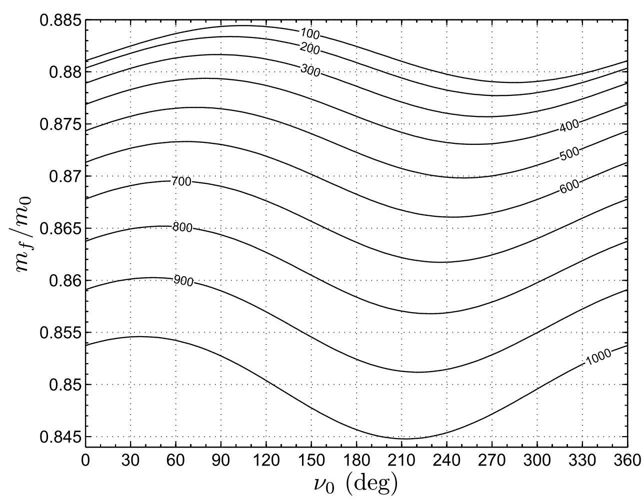

Fig. 3 Mass ratio, parametrized with m_0 (kg), as a function of the initial true anomaly of Earth. The term m_f is the spacecraft mass before the application of the impulsive ΔV.

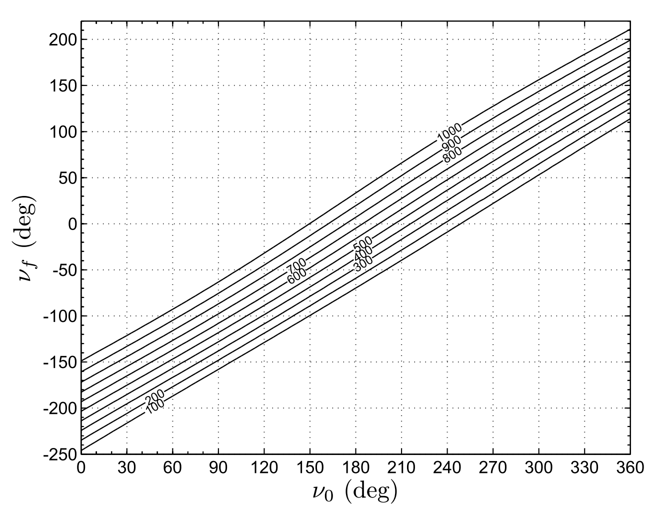

Fig. 4 Final spacecraft position, parametrized with m_0 (kg), as a function of the initial true anomaly of Earth.

cone angle is equal to $\alpha_{\max}$ when the thruster is on ($\tau = 1$). In that case, $\alpha = \alpha_{\max} =$ constant amounts to considering constant both the thrust modulus and the mass flow rate. In the simulations the value $\alpha_{\max} = 4.58$ deg has been used [4]. The corresponding values for the thrust modulus is 1.0032 N and the mass flow rate is $\beta = 0.5$ kg/day.

The assumption $\alpha = \alpha_{\max}$ is strengthened by all the simulations performed in the two-dimensional case [4,5]. The results confirm that the azimuthal thrust component corresponding to the maximum available cone angle allows the spacecraft to increase the azimuthal velocity and, therefore, to reduce the velocity change required for the rendezvous. This behavior still holds (as confirmed by simulation) in a three-dimensional transfer. In fact, when orbits having a relative inclination are considered, the azimuthal thrust component is effective not only for varying the spacecraft azimuthal velocity, but also for changing the orbital inclination.

The results have been parametrized with the initial mass m_0 in the range [100, 1000] kg and summarized in Figs. 1–3. The numerical results are representative of spacecraft having an initial propulsive acceleration in the range [1, 10] mm/s^2.

The results show significant differences with respect to a two-dimensional circle-to-circle transfer problem [4]. In fact, both the mission time and the required ΔV in the two-dimensional case represent a sort of mean value for the corresponding three-dimensional transfer problem. Remarkable oscillations of Δt (on the order of 12–16%) and of ΔV (on the order of 10%) are present when the spacecraft initial position is varied on the Earth's orbit. For example, assuming a spacecraft launch mass equal to 1000 kg, the two-dimensional solution would estimate a ΔV required to circularize the orbit equal to 6.65 km/s, whereas the three-dimensional simulation suggests that the true value varies in the range [6.3, 7.0] km/s depending on the starting value of ν_0. Notably, the simulation outputs can be used to determine the launch windows for a given mission by comparing the positions of Earth and Jupiter (given in terms of true anomaly) with the true planetary ephemerides. The final spacecraft position is shown in Fig. 4 as a function of the initial true anomaly of Earth.

Although trajectories toward outer planets can be obtained with the outlined methodology, preliminary analyses performed by Funaki et al. [3] have shown that the chance of reaching more distant planets, such as Saturn and Uranus, is not currently realistic. In fact, for such missions the tilt angle of the dipole axis should reach at least 10 deg [3], a value which is outside the near-term M2P2 technology.

Conclusions

The problem of outer planet missions based on minimagnetospheric plasma propulsion has been investigated. The optimal control law that minimizes the impulsive velocity change required to accomplish the rendezvous between the spacecraft and the target has been derived using an indirect method. The real shape and space orientation of both the departure and arrival planetary orbits has been considered. Using the true final anomaly of the target planet as an output of the optimization process, the global minimum solutions for an Earth-target three-dimensional transfer are obtained and the corresponding launch windows are calculated. A parametric study with different values of the spacecraft mass has been discussed to point out the impact of the mass budget on the mission characteristics. The effectiveness of the control law has been shown by simulating interplanetary rendezvous missions to Jupiter. The simulations reveal significant differences with respect to the results achievable using a two-dimensional trajectory model.

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